Practice Test - Chapter 5

Simplify.

1. $(3a)^2(7b)^4$

SOLUTION:

$$(3a)^2(7b)^4 = \left(3^2a^2\right)\left(7^4b^4\right) \quad \text{Power of a Product}$$
$$= \left(9a^2\right)\left(2401b^4\right) \quad \text{Simplify.}$$
$$= 21{,}609a^2b^4 \quad \text{Simplify.}$$

2. $(7x - 2)(2x + 5)$

SOLUTION:

$$(7x-2)(2x+5) = 14x^2 + 35x - 4x - 10 \quad \text{FOIL}$$
$$= 14x^2 + 31x - 10 \quad \text{Combine like terms.}$$

3. $(2x^2 + 3x - 4) - (4x^2 - 7x + 1)$

SOLUTION:

$$\left(2x^2 + 3x - 4\right) - \left(4x^2 - 7x + 1\right)$$
$$= 2x^2 + 3x - 4 - 4x^2 + 7x - 1 \quad \text{Distributive property}$$
$$= -2x^2 + 10x - 5 \quad \text{Simplify.}$$

4. $(4x^3 - x^2 + 5x - 4) + (5x - 10)$

SOLUTION:

$$\left(4x^3 - x^2 + 5x - 4\right) + (5x - 10)$$
$$= 4x^3 - x^2 + 5x - 4 + 5x - 10 \quad \text{Simplify.}$$
$$= 4x^3 - x^2 + 10x - 14 \quad \text{Combine like terms.}$$

5. $(x^4 + 5x^3 + 3x^2 - 8x + 3) \div (x + 3)$

SOLUTION:

$$\left(x^4 + 5x^3 + 3x^2 - 8x + 3\right) \div (x + 3)$$
$$= \frac{x^4 + 5x^3 + 3x^2 - 8x + 3}{x + 3} \quad \text{Write as a fraction.}$$
$$= \frac{(x+3)\left(x^3 + 2x^2 - 3x + 1\right)}{(x+3)} \quad \text{Factor numerator.}$$
$$= x^3 + 2x^2 - 3x + 1 \quad \text{Simplify.}$$

6. $(3x^3 - 5x^2 - 23x + 24) \div (x - 3)$

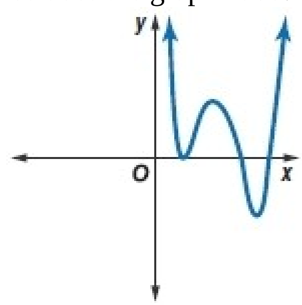
SOLUTION:

Use long division.

$$\begin{array}{r} 3x^2 + 4x - 11 \\ x - 3\overline{)\,3x^3 - 5x^2 - 23x + 24} \\ \underline{(-)\ 3x^3 - 9x^2} \\ 4x^2 - 23x \\ \underline{(-)\ 4x^2 - 12x} \\ -11x + 24 \\ \underline{(-)\ -11x + 33} \\ -9 \end{array}$$

The remainder is $-\frac{9}{x-3}$ so $\left(3x^3 - 5x^2 - 23x + 24\right) \div (x - 3) = 3x^2 + 4x - 11 - \frac{9}{x-3}$.

7. **MULTIPLE CHOICE** How many unique real zeros does the graph have?



A 0
B 2
C 3
D 5

SOLUTION:

The graph intersects the *x*-axis in three points. Therefore, it has three unique real zeros. Option C is the correct answer.

8. If $c(x) = 3x^3 + 5x^2 – 4$, what is the value of $4c(3b)$?

SOLUTION:

Substitute $3b$ for x in the function and simplify.

$$\begin{aligned}c(3b) &= 3(3b)^3 + 5(3b)^2 - 4\\ &= 3(27b^3) + 5(9b^2) - 4\\ &= 81b^3 + 45b^2 - 4\end{aligned}$$

Multiply the function $c(3b)$ by 4.

$$\begin{aligned}4c(3b) &= 4(81b^3 + 45b^2 - 4)\\ &= 324b^3 + 180b^2 - 16\end{aligned}$$

Complete each of the following.
a. Graph each function by making a table of values.
b. Determine consecutive integer values of *x* between which each real zero is located.
c. Estimate the *x*-coordinates at which the relative maxima and relative minima occur.

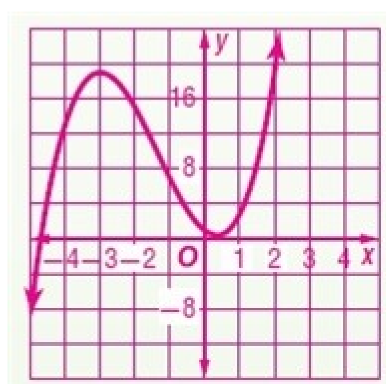
9. $g(x) = x^3 + 4x^2 – 3x + 1$

SOLUTION:

a. Substitute –5, –4, –3, –2, –1, 0, 1, 2 and 3 for x and make the table.

x	$g(x)$	(x, y)
–5	$g(-5) = (-5)^3 + 4(-5)^2 - 3(-5) + 1$ $= -9$	$(-5, -9)$
–4	$g(-4) = (-4)^3 + 4(-4)^2 - 3(-4) + 1$ $= 13$	$(-4, 13)$
–3	$g(-3) = (-3)^3 + 4(-3)^2 - 3(-3) + 1$ $= 19$	$(-3, 19)$
–2	$g(-2) = (-2)^3 + 4(-2)^2 - 3(-2) + 1$ $= 15$	$(-2, 15)$
–1	$g(-1) = (-1)^3 + 4(-1)^2 - 3(-1) + 1$ $= 7$	$(-1, 7)$
0	$g(0) = (0)^3 + 4(0)^2 - 3(0) + 1$ $= 1$	$(0, 1)$
1	$g(1) = (1)^3 + 4(1)^2 - 3(1) + 1$ $= 3$	$(1, 3)$
2	$g(2) = (2)^3 + 4(2)^2 - 3(2) + 1$ $= 19$	$(2, 19)$
3	$g(3) = (3)^3 + 4(3)^2 - 3(3) + 1$ $= 55$	$(3, 55)$

Plot the points and connect the points as shown in the figure.



b. The graph intersects the x-axis between –5 and –4.

c. The relative maximum at $x = –3$.
The relative minimum at $x \approx 0.3$.

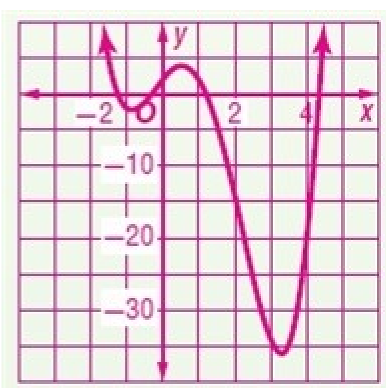
10. $h(x) = x^4 - 4x^3 - 3x^2 + 6x + 2$

SOLUTION:

a. Substitute –2, –1, 0, 1, 2, 3, 4 and 5 for x and make the table.

x	$h(x)$	(x, y)
–2	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= 26$	$(-2, 26)$
–1	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= -2$	$(-1, -2)$
0	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= 2$	$(0, 2)$
1	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= 2$	$(1, 2)$
2	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= -14$	$(2, -14)$
3	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= -34$	$(3, -34)$
4	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= -22$	$(4, -22)$
5	$h(-3) = (-3)^4 - 4(-3)^3 - 3(-3)^2 + 6(-3) + 2$ $= 82$	$(5, 82)$

Plot the points and connect the points as shown in the figure.



b. The graph intersects the x-axis between –2 and –1, between –1 and 0, between 1 and 2, between 4 and 5.

c. The relative maximum at $x \approx 0.5$.

The relative minimum at $x \approx -0.8$ and $x \approx 3.3$.

Factor completely. If the polynomial is not factorable, write *prime*.

11. $8y^4 + x^3y$

SOLUTION:

$$8y^4 + x^3y$$
$$= y\left(8y^3 + x^3\right) \quad \text{Factor the GCF.}$$
$$= y\left((2y)^3 + x^3\right) \quad \text{Replace } 8y^3 \text{ with } (2y)^3.$$
$$= y(2y + x)\left(4y^2 - 2xy + x^2\right) \quad \text{Simplify.}$$

12. $2x^2 + 2x + 1$

SOLUTION:

The polynomial cannot be factored so it is prime.

13. $a^2x + 3ax + 2x - a^2y - 3ay - 2y$

SOLUTION:

$$a^2x + 3ax + 2x - a^2y - 3ay - 2y$$
$$= \left(a^2x + 3ax + 2x\right) + \left(-a^2y - 3ay - 2y\right) \quad \text{Group to find a GCF.}$$
$$= x\left(a^2 + 3a + 2\right) - y\left(a^2 + 3a + 2\right) \quad \text{Factor.}$$
$$= (x - y)\left(a^2 + 3a + 2\right) \quad \text{Distributive Property}$$
$$= (x - y)(a + 1)(a + 2) \quad \text{Factor.}$$

Solve each equation.

14. $8x^3 + 1 = 0$

SOLUTION:

$$8x^3 + 1 = 0 \quad \text{Original equation}$$
$$(2x)^3 + 1^3 = 0 \quad \text{Rewrite as sum of 2 cubes.}$$
$$(2x + 1)\left(4x^2 - 2x + 1\right) = 0 \quad a^3 + b^3 = (a + b)(a^2 - ab + b^3)$$

By Zero Product Property:

$$4x^2 - 2x + 1 = 0 \quad \text{or} \quad 2x + 1 = 0$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(1)}}{2(4)} \quad \text{or} \quad 2x = -1$$
$$x = \frac{2 \pm \sqrt{4 - 16}}{8} \quad \text{or} \quad x = -\frac{1}{2}$$
$$x = \frac{1 \pm i\sqrt{3}}{4} \quad \text{or} \quad x = -\frac{1}{2}$$

The solutions are $-\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}$.

15. $x^4 - 11x^2 + 28 = 0$

SOLUTION:

$$x^4 - 11x^2 + 28 = 0$$

Let $y = x^2$.

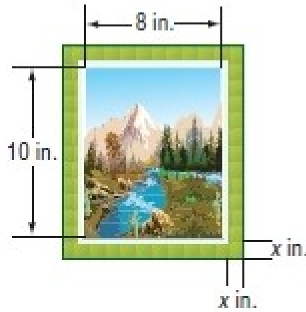
$$y^2 - 11y + 28 = 0 \quad \text{Original equation}$$
$$(y-7)(y-4) = 0 \quad \text{Factor.}$$

By Zero Product Property:

$$\begin{array}{lll} y-7=0 & \text{or} & y-4=0 \\ y=7 & \text{or} & y=4 \\ x^2=7 & \text{or} & x^2=4 \\ x=\pm\sqrt{7} & \text{or} & x=\pm 2 \end{array}$$

The solutions are $\pm\sqrt{7}, \pm 2$.

16. **FRAMING** The area of the picture and frame shown below is 168 square inches. What is the width of the frame?



SOLUTION:

The area of the picture and frame in terms of x is $(10+2x)(8+2x)$.

$$\begin{array}{rl} (10+2x)(8+2x)=168 & \text{Write an equation.} \\ 80+20x+16x+4x^2=168 & \text{FOIL} \\ 4x^2+36x+80-168=0 & \text{Combine like terms.} \\ 4x^2+36x-88=0 & \text{Combine like terms.} \\ 4\left(x^2+9x-22\right)=0 & \text{Factor out GCF..} \\ \Rightarrow x^2+9x-22=0 & \text{Zero Product Property} \end{array}$$

Solve the equation $x^2 + 9x - 22 = 0$.

$$x^2 + 9x - 22 = 0$$
$$(x-2)(x+11) = 0$$
$$x = 2 \text{ or } x = -11$$

The value of x cannot be negative. Therefore, the width of the frame is 2 in.

17. **MULTIPLE CHOICE** Let $f(x) = x^4 - 3x^3 + 5x - 3$. Use synthetic substitution to find $f(-2)$.
 F 7
 G 7
 H 33
 J 21

SOLUTION:
$f(-2)$ is the remainder when the polynomial $f(x)$ divided by $x + 2$.

–2	1	–3	0	5	–3
	0	–2	10	–20	30
	1	–5	10	–15	27

Therefore, the remainder is 27.
That is, $f(-2) = 27$.
Option G is the correct answer.

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

18. $2x^3 + 15x^2 + 22x - 15$; $x + 5$

SOLUTION:
Use Synthetic Division to complete the problem.

–5	2	15	22	–15
	0	–10	–25	15
	2	5	–3	0

That is $(2x^3 + 15x^2 + 22x - 15) \div (x + 5) = (2x^2 + 5x - 3)$.

$2x^2 + 5x - 3 = (2x - 1)(x + 3)$

Therefore, the remaining factors are $(x + 3)$ and $(2x - 1)$.

19. $x^3 - 4x^2 + 10x - 12$; $x - 2$

SOLUTION:
Use Synthetic Division to complete the problem.

2	1	–4	10	–12
	0	2	–4	12
	1	–2	6	0

That is $(x^3 - 4x^2 + 10x - 12) \div (x - 2) = (x^2 - 2x + 6)$.

Therefore, the remaining factor is $x^2 - 2x + 6$.

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

20. $p(x) = x^3 - x^2 - x - 3$

SOLUTION:

$$p(x) = x^3 - x^2 - x - 3$$

There is a sign change for the coefficient of $p(x)$. So, the function has 1 or zero positive real zeros.

$$p(-x) = -x^3 - x^2 + x - 3$$

There are 2 sign changes for the coefficient of $p(-x)$. So, the function has 2 or zero negative real zeros. Therefore, there are 1 positive, 2 or 0 negative real zeros, 2 or 0 imaginary zeros.

21. $p(x) = 2x^6 + 5x^4 - x^3 - 5x - 1$

SOLUTION:

$$p(x) = 2x^6 + 5x^4 - x^3 - 5x - 1$$

There is a sign change for the coefficient of $p(x)$. So, the function has 1 positive real zeros.

$$p(-x) = 2x^6 + 5x^4 + x^3 + 5x - 1$$

There is a sign changes for the coefficient of $p(-x)$. So, the function has 1 or 0 negative real zeros.
Since the function has degree 6, it will have 6 zeros.
The function has 2 real zeros, so it will have 4 imaginary zeros.

Therefore, there are 1 positive, 1 negative real zeros, 4 imaginary zeros.

Find all zeros of each function.

22. $p(x) = x^3 - 4x^2 + x + 6$

SOLUTION:

$$p(-x) = -x^3 - 4x^2 - x + 6$$

There is a sign changes for the coefficient of $p(-x)$. So, the function has 1 negative real zeros.

$$p(x) = x^3 - 4x^2 + x + 6$$

There are 2 sign change for the coefficient of $p(x)$. So, the function has 2 or 0 positive real zeros.
Since the function has degree 3, it will have 3 zeros.
The function has 1 or 3 real zeros, so it will have 2 or 0 imaginary zeros.

Therefore, $p(x)$ has 2 positive and 1 negative real zeros or 1 negative and 2 imaginary zeros.

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & 0 & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & \underline{|0} \end{array}$$

$$\begin{aligned} p(x) &= x^3 - 4x^2 + x + 6 \\ &= (x+1)(x^2 - 5x + 6) \\ &= (x+1)(x-2)(x-3) \end{aligned}$$

Substitute 0 for $p(x)$ and solve for x.

$$\begin{array}{llll} x+1=0 \text{ or} & x-2=0 & \text{or} & x-3=0 \\ x=-1 \quad \text{or} & x=2 & \text{or} & x=3 \end{array}$$

Therefore, the zeros of the function are –1, 2 and 3.

23. $p(x) = x^3 + 2x^2 + 4x + 8$

SOLUTION:

$$p(-x) = -x^3 + 2x^2 - 4x + 8$$

There are 3 sign changes for the coefficient of $p(-x)$. So, the function has 3 or 1 or zero negative real zeros.

$$p(x) = x^3 + 2x^2 + 4x + 8$$

There is no sign change for the coefficient of $p(x)$. So, the function has 0 positive real zeros. Therefore, $p(x)$ has 3 negative real zeros or 1 negative and 2 imaginary zeros.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 4 & 8 \\ & 0 & -2 & 0 & -8 \\ \hline & 1 & 0 & 4 & \underline{|0} \end{array}$$

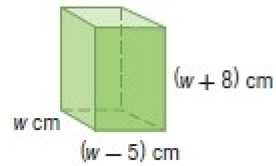
$$\begin{aligned} p(x) &= x^3 + 2x^2 + 4x + 8 \\ &= (x+2)(x^2+4) \end{aligned}$$

Substitute 0 for $p(x)$ and solve for x.

$$\begin{array}{lcl} x^2 + 4 = 0 & \text{or} & x + 2 = 0 \\ x^2 = -4 & \text{or} & x = -2 \\ x = \pm 2i & \text{or} & x = -2 \end{array}$$

Therefore, the zeros of the function are $-2i$, $2i$ and -2.

24. **GEOMETRY The volume of the rectangular prism shown is 612 cubic centimeters. Find the dimensions of the prism.**



SOLUTION:

The volume of the rectangular prism is $w(w+8)(w-5)$.

Therefore,

$$\begin{aligned} w(w+8)(w-5) &= 612 \\ w(w^2 + 3w - 40) &= 612 \\ w^3 + 3w^2 - 40w - 612 &= 0 \\ (w-9)(w^2 + 12w + 68) &= 0 \end{aligned}$$

By Zero Product Property:

$$\begin{array}{lcl} w - 9 = 0 & \text{or} & w^2 + 12w + 68 = 0 \\ w = 9 & \text{or} & w = \dfrac{-12 \pm \sqrt{-128}}{2} \\ w = 9 & \text{or} & w = -6 \pm 4i\sqrt{2} \end{array}$$

The length should be real. Therefore, $w = 9$, $w + 8 = 17$, and $w - 5 = 4$.

The dimensions of the rectangular prism are 9cm by 17 cm by 4 cm.

25. List all possible rational zeros of $f(x) = 2x^4 + 3x^2 - 12x + 8$.

SOLUTION:

The factors of the constant term are 1, 2, 4 and 8. The factors of the leading coefficient are 1 and 2. Therefore, all possible rational zeros are $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$.